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Auction design and favoritism*

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The theory of auctions has ignored the fact that often auction designers, not the principal, design auctions. In a multi attribute auction, the auction designer may bias his subjective evaluation of quality or distort the relative weights of the various attributes to favor a specific bidder, an ancient concern in the procurement of weapons, in the auctioning of government contracts and in the purchase of electricity by regulated power companies. The paper analyzes the steps to be taken to reduce the possibility of favoritism. It is first shown that in the absence of favoritism, quality differentials among firms are more likely to be ignored, if the auction designer has imperfect information about the firm's costs. Second, if the auction designer may collude with only one bidder, the other bidders should be chosen if they are at least as efficient as the former bidder, and no hard information about quality differentials is released by the auction designer that would justify fair discrimination in favor of the former bidder. Last, if the auction designer can collude with any bidder, the optimal auction tends to a symmetric auction in which quality differentials are ignored. The possibility of favoritism reduces the auction designer's discretion and makes the selection process focus on non-manipulable (monetary) dimensions of bids.

1. Introduction

The economic theory of auctions[1] has analyzed the design of bidding procedures that maximize the principal's expected revenue. It has ignored the fact that the auction designer is in general not the principal, but its agent: An auction house's duty is to sell at the best terms for the principal; a contractor may select a subcontractor on behalf of the buyers; and the Department of Defense acts as an agent for Congress or the public when soliciting and evaluating offerors' proposals for weapons acquisitions. There has been much

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[1]Recent surveys include McAfee–McMillan (1987) and Milgrom (1987).

concern that the auction designer may prefer or collude with a specific buyer. And indeed most military or governmental markets acquisition regulations[2] go at great length to impose rules aimed at curbing favoritism. Similarly, the European Economic Commission, alarmed by the abnormally large percentage (above 95% in most countries) of government contracts awarded to domestic firms is trying to design rules that would foster fairer competition between domestic and foreign suppliers and would fit better than recent experience with the aim of fully opening borders in 1992.

In our view, the importance of the threat of collusion between auction designers and specific bidders depends much on what is being auctioned off. When the object of bidding is simple, as is often the case in the auction house example, the principal (the seller) may conciliate the goal that the auctioneer enjoys little discretion and that the good be sold at the best terms; this results from the fact that under some circumstances,[3] the seller's expected revenue is maximized by auction procedures (first- or second-bid auctions) that requires no decentralized information and therefore can be perfectly controlled by the principal.

The procurement examples illustrate the possibility that the stake of bidding be multidimensional (in the case of a good for sale, the seller is in general interested only in the price dimension). An incentive scheme to realize a given project includes at least a fixed fee and a coefficient of cost sharing by the principal. Furthermore the principal generally cares about other attributes of the trade with the winning bidder: quality and reliability of service, date of delivery, probability of bankruptcy of the supplier, reputation for fairness and competency in dealing with contingencies not

[2]See, e.g., the U.S. Air Force Regulation 70-15 or the *Instruction pour l'Application du Code des Marchés Publics* (Journal Officiel de la République Française, 1976). Constraints on acquisition procedures have a long history. For instance, the early twentieth century State and federal regulations in the U.S. required that gas and electric utilities and some agencies (e.g. ICC) secure competitive bids for their purchases.

[3]We are here alluding to the revenue equivalence theorem [Vickrey (1961), Myerson (1981)]. If the bidders' valuations for the good are private, independent and are drawn from the same distribution, and if the bidders are risk neutral, the first- and second-bid auctions maximize the seller's expected revenue. In this respect, it is interesting to note that the detailed procedures of the U.S. Air Force Regulation 70-15 'do not apply if the contract is awarded primarily on the basis of price competition'. When the valuations exhibit common values, or when the bidders are asymmetric or risk averse, such simple auctions are no longer optimal [Milgrom–Weber (1982), Maskin–Riley (1984)]. For instance, under some regularity conditions, more eager buyers should be discriminated against [Myerson (1981)]; if the auction designer has private information about who is more eager to buy, phenomena such as the ones described in this paper may arise. Last we do not claim that first- and second-bid auctions are completely immune to collusion between the agency and specific bidders (for instance many regulations that specify that a contract be awarded at the lowest price offer ensure that no communication of the maximum price or of the competitors' bids, or of secret information held by the principal to a specific bidder occurs); rather such auctions are collusion proof under some circumstances and when they are not, the scope of collusion is relatively limited.

foreseen by the contract, level of pollution associated with the production by this specific firm, etc. This raises two related concerns. First, the contract designer must assign relative weights to the observable characteristics of the bids, i.e., determine the monetary values of units of some dimensions of performance; and the optimal choice of weights is likely to depend on information held by the contract designer. Second, some of these characteristics may not be observable by the principal and must be assessed by the contract designer. In both cases, the information held by the contract designer about the principal's optimal source selection may give rise to collusion between the contract designer and some bidders.[4] By choosing weights appropriately or by misrepresenting the quality of their projects, the auction designer may favor one firm over the others.[5] We will say that the auction designer engages in *unfair* discrimination.

The purchase of power by U.S. electric utilities from qualifying cogeneration and small power production facilities is a good case in point. In their interpretation of the 1978 PURPA act, many States have forced electric utilities to use competitive bidding procedures to purchase power rather than buy internally. A typical request for proposal (RFP) specifies a fixed quantity to be supplied (number of megawatts) and contains a detailed scoring system for proposals. For each bid, a score is given for each broad category (itself an aggregation of more detailed attributes): price factor; 'system optimization factor' (location of facilities, maintenance, power for the utility to dispatch, i.e. to have operating control over the amount and the timing of electricity supplies by the qualifying facility, ...); 'economic confidence factor' (probability of bankruptcy and financial structure of the qualifying facility, ...); 'project development factor' (technical characteristics, experience of seller, ...); etc. The weights among the different factors are fixed in advance in the RFP. While the States imposed competitive bidding on electric utilities, the latter have kept substantial discretion despite the seemingly objective scoring systems. First, the weights among various factors may vary substantially. For instance, Virginia Power puts weight 70% on economic factors while Boston Edison puts less weight on such factors (25% on 'price factor', plus some weight put on quasi-monetary factors such as dispatchability). Second, the utility evaluates the levels of subjective characteristics such as the reputation or probability of bankruptcy of the qualifying facilities, or the value of dispatching rights (which depends on the utility's own resources, and on

[4]Our model is one of unobservable quality, and not one of weights to be determined; but the same principles seem to apply to both situations.

[5]The potential discretion of contract designers appears clearly in the vague objectives set up by acquisition regulations: For instance 'the principal objective of the major source selection process is to select the source whose proposal has the highest degree of credibility and whose performance can be expected to best meet the government's requirements at an affordable cost' (U.S. Air Force regulation 70-15, p. 3).

other bids if the contract is shared among several qualifying facilities).[6] Very similar observations can be made concerning the scoring systems used by the Department of Defense.[7]

This paper is a first exploration of the control of auction designers by principals.[8] Section 2 sets up the model. Two suppliers, the 'agents', compete for a procurement contract for the 'principal' (a government or a Commission of the European Community). A contract specifies a monetary transfer to the winning agent and an obligation to reach a cost target. An agency, the 'supervisor', has more information than the principal about the social surplus, henceforth 'quality', brought about by each potential supplier. One can think of 'quality' as reflecting the quality of the supplier's output, its probability of bankruptcy or the likelihood of being fair in unforeseen contingencies. We first assume that the supervisor is benevolent (truthfully reveals its information, if any, to the principal) and that the firm's technologies are commonly known. The principal then compares the quality differential and the cost differential between the agents. Depending on the parameters, the cost differential or the quality differential may be 'decisive' in the principal's selection (if each firm has an advantage in one dimension and a disadvantage in the other. If both criteria agree, the choice between the agents is trivial).

We next relax the assumption that the firms' technologies are commonly known. If firms have private information about their costs, the cost differential is more likely to be decisive (section 3). This result can be explained as follows. To limit the firms' informational rents, the principal reduces the power of incentive schemes for intrinsically high-cost types. This lowers their cost-reducing activity and increases the realized cost differentials. Another way of putting it is that, by favoring cost over quality, the principal reduces the probability that a high-cost firm be chosen and thus the temptation for a low-cost firm to pretend that its cost is intrinsically high.

Last, the paper also relaxes the assumption that the supervisor is benevolent and does not collude with bidders. The potential for collusion stems from the agents' stake in the supervisor's report about quality (they enjoy a rent from their technological knowledge if selected). When the

[6]In some States, the utility retains one more degree of freedom. For instance, New York State Electric and Gas Corporation uses a scoring system only to select an 'initial award group'. The utility uses its judgment to select among the screened sellers, in order to 'maintain flexibility' (Executive Summary, page 2). The utility can reject any or all proposals, and can consider a substi[illegible]n in favor of 'non-bid alternatives' (including construction of a plant by the utility itself).

[7]The DOD's RFPs put scores on price, schedules, logistics, management, past experience, technological characteristics (e.g., range, maneuverability, take-off/landing distance, cruise speed, for an airplane), etc.

[8]In most of the paper, we ignore the important issue of collusion among bidders. For analyses of bid rigging, see Graham–Marshall (1987), McAfee–MacMillan (1988) and Mailath–Zemsky (1989). See also section 4.

supervisor's information about quality is 'soft' (i.e., is not verifiable by the principal), the principal imposes a symmetric auction even though the supervisor's information about quality would vindicate discrimination between the two bidders (section 4).

The analysis of the case of 'hard' information (information that is verifiable if communicated to the principal) is more difficult. We carry it first only in the special case in which the agency can collude only with one bidder (section 5). This assumption may be appropriate for auctions between a domestic and a foreign firm; the supervisor (the domestic government or agency in this application) may be able to trade favors with the domestic firm but not with the foreign firm.[9] The principal (the European Community) relies on the supervisor for the provision of hard information (about the quality or fit of the agents with the needs) giving reasons to discriminate between the 'domestic agent' and the 'foreign agent'.[10] The main conclusion is that by favoring the foreign firm when no information about quality is disclosed, no welfare loss is imposed on the principal by the threat of collusion.

The case of symmetric collusion is taken up in section 6 where only an exploratory analysis is provided as developing techniques to study collusion with several informed parties is outside the scope of this paper. We find that two cases must be considered. If the quality differentials are high enough collusion proofness is ensured by appropriately motivating the agency and the auction is similar to that in section 3 but with weaker incentive schemes. If the quality differentials are low, the agency faces a flat incentive scheme, and the stakes in collusion are reduced by altering the auction towards a more symmetric auction and by decreasing the power of incentive schemes for firms.

2. The model

For simplicity, we assume that only two firms[11] can participate in the auction. Each firm $i, i=1,2$, is able to realize an indivisible public project at cost:

[9]A similar situation may arise in the case of a division of a firm choosing between an internal and an external supplier, or in the case of a department choosing between an insider and an outsider for a tenured position in a given field.

[10]We here take the view that the European Community has the power to dictate auctioning procedures or to ex post punish governments if these are not respected. This assumption has proved unrealistic in spite of the 1971 and 1976 directives to create a 'Europe of governmental markets'; but the European Community is currently studying how to regulate governmental contracts in a more effective way than in the past. We should also note that the Court of Justice and the European Community have means of enforcing the directives of fair competition: legal procedures, cancellation of financial loans or of subsidies, etc.

[11]See Laffont–Tirole (1989) for an analysis of the regulation of a natural monopoly's quality.

$$C^i = \beta^i - e^i, \qquad i = 1, 2,$$

where β^i is firm i's efficiency parameter and e^i is manager i's effort level (which is incurred only if this firm is selected).

The firms' efficiency parameters are independently drawn from a common-knowledge two-point probability distribution on $\{\underline{\beta}, \bar{\beta}\}$. Let $\nu = \Pr(\beta^i = \underline{\beta})$ and $\Delta\beta = \bar{\beta} - \underline{\beta}$.

Manager $i, i = 1, 2$ has utility function

$$U^i = t^i - \psi(e^i), \qquad i = 1, 2,$$

where t^i is the net (i.e. in addition to the reimbursement of cost) monetary transfer that he receives from the regulator and $\psi(e^i)$ is his disutility of effort with $\psi' > 0$, $\psi'' > 0$, $\psi''' \geqq 0$. Moreover, each firm's outside opportunity level (individual rationality; IR) is normalized at 0.

The consumers' valuation of the project can take one of two values $(\bar{S}, \underline{S})$ with $\bar{S} > \underline{S}$ according to the quality of the firm. S^i denotes the valuation when firm i realizes the project. Again to simplify the analysis we assume that either $S^1 = \bar{S}$, $S^2 = \underline{S}$ or $S^1 = \underline{S}$, $S^2 = \bar{S}$ and that $\Pr(S^1 = \bar{S}, S^2 = \underline{S}) = 1/2$. We will refer to the firm with the $\bar{S}$ value as the high quality firm. Let $\Delta S = \bar{S} - \underline{S}$.

These values of the project cannot be contracted upon, but ex ante an agency may learn these values. We assume that the agency can be in one of three states of information σ:

$$\sigma = 1 \Leftrightarrow S^1 = \bar{S};\ S^2 = \underline{S},$$

$$\sigma = 2 \Leftrightarrow S^1 = \underline{S},\ S^2 = \bar{S},$$

$$\sigma = 0 \Leftrightarrow \emptyset.$$

In state $\emptyset$, the agency learns nothing and let $\xi = \Pr(\sigma = 1) = \Pr(\sigma = 2) \leqq 1/2$.

The agency receives income s from the principal, has utility function $V(s) = s$ for $s \geqq s^*$ and its ex post utility level cannot be lower than s^*.

The principal's objective function is the sum of welfares in society. Its ex post value is

$$W = S - (1+\lambda)(C + t + s) + U + (s - s^*) = S - (1+\lambda)(C + \psi(e))$$

$$- \lambda(U + (s - s^*)) - (1+\lambda)s^* \qquad \text{where } \lambda > 0$$

is the social cost of public funds, i.e., the shadow cost for the principal to raise money through distortionary taxation; t is the total transfer to firms; U

the sum of the firms' utilities and C and e the cost and effort of the selected firm.

Full information: As a benchmark case we derive the optimal regulatory scheme for a utilitarian principal when a benevolent agency knows σ, the values of the β^i and can observe costs. Let $x^i_\sigma(\beta^1, \beta^2)$ denote the probability of selecting firm i in the state of information σ for the values β^1 and β^2 of the efficiency parameters.

We must distinguish two cases to determine the optimal values of $x^i_\sigma(\cdot)$:

Case 1 $\Delta S \leqq (1+\lambda)\Delta\beta$.

This condition means that choosing the more efficient firm is more important than choosing the better quality firm. We will say that 'cost considerations are decisive', Straightforward reasoning shows that:

$$x^1_1(\underline{\beta},\bar{\beta})=1, \quad x^1_1(\bar{\beta},\underline{\beta})=0, \quad x^1_1(\underline{\beta},\underline{\beta})=x^1_1(\bar{\beta},\bar{\beta})=1,$$

$$x^1_2(\underline{\beta},\bar{\beta})=1, \quad x^1_2(\bar{\beta},\underline{\beta})=0, \quad x^1_2(\underline{\beta},\underline{\beta})=x^1_2(\bar{\beta},\bar{\beta})=0,$$

$$x^1_0(\underline{\beta},\bar{\beta})=1, \quad x^1_0(\bar{\beta},\underline{\beta})=0, \quad x^1_0(\underline{\beta},\underline{\beta}) \text{ and } x^1_0(\bar{\beta},\bar{\beta})$$

are indeterminate in [0, 1].

That is, the low cost firm is always selected. At equal cost, the better quality firm is selected; and if there is no information about quality, any random selection will do.

The social cost of the project is

$$(1+\lambda)(\beta-e+\psi(e)).$$

Effort minimizes cost if $\psi'(e)=1$ or $e=e^*$.

For a utilitarian principal, optimal regulation leads to $e=e^*$ and to the $x^i_\sigma(\cdot)$ function defined above. Accordingly expected social welfare is:

$$W_1^{FB}=2\xi(\bar{S}-\nu(1-\nu)\Delta S)+(1-2\xi)\left(\frac{\bar{S}+\underline{S}}{2}\right)$$

$$-(1+\lambda)(\underline{\beta}-e^*+\psi(e^*)+s^*)-(1-\nu)^2(1+\lambda)\Delta\beta.$$

Case 2 $\Delta S > (1+\lambda)\Delta\beta$.

We will say that 'quality considerations are decisive'. Straightforward reasoning shows that:

$$x_1^1(\underline{\beta},\bar{\beta})=x_1^1(\bar{\beta},\underline{\beta})=x_1^1(\underline{\beta},\underline{\beta})=x_1^1(\bar{\beta},\bar{\beta})=1,$$

$$x_2^1(\underline{\beta},\bar{\beta})=x_2^1(\bar{\beta},\underline{\beta})=x_2^1(\underline{\beta},\underline{\beta})=x_2^1(\bar{\beta},\bar{\beta})=0,$$

$$x_0^1(\underline{\beta},\bar{\beta})=1,\quad x_0^1(\bar{\beta},\underline{\beta})=0,\quad x_0^1(\underline{\beta},\underline{\beta}) \text{ and } x_0^1\ (\bar{\beta},\bar{\beta})$$

are indeterminate in $[0,1]$.

As in case 1 we define welfare:

$$W_2^{FB}=2\xi\bar{S}+(1-2\xi)\left(\frac{\bar{S}+\underline{S}}{2}\right)-(1+\lambda)(\underline{\beta}-e^*+\psi(e^*)+s^*)$$

$$-(1+\lambda)[2\xi(1-\nu)+(1-2\xi)(1-\nu)^2]\Delta\beta.$$

3. Optimal regulation with a benevolent agency

In this section we maintain the assumption that the agency is benevolent (does not collude, i.e. truthfully reveals its information to the principal), but we recognize the asymmetry of information between the agency and the firms concerning the efficiency parameters. Specifically, β^i is known to firm i only and costs are ex post observable by the agency.

For each state σ of its information the agency organizes an auction of contracts. From the revelation principle we know that such an auction is equivalent to a revelation mechanism.

For each value of σ, let $\{t_\sigma^1(\beta^1,\beta^2),\ C_\sigma^1(\beta^1,\beta^2),\ t_\sigma^2(\beta^1,\beta^2),\ C_\sigma^2(\beta^1,\beta^2),\ x_\sigma^1(\beta^1,\beta^2),\ x_\sigma^2(\beta^1,\beta^2)\}$ be a revelation mechanism which specifies transfers to firm i, $t_\sigma^i(\beta^1,\beta^2)$, a cost target for firm i if selected $C_\sigma^i(\beta^1,\beta^2)$ and a probability of selecting firm i, $x_\sigma^i(\beta^1,\beta^2)\in[0,1]$ for each announcement β^1,β^2 of cost characteristics. Under the natural monopoly assumption, $x_\sigma^1+x_\sigma^2\leqq 1$ (and, at the optimum $x_\sigma^1+x_\sigma^2=1$ if the surpluses are sufficiently large, which we will assume).

Incentive compatibility in the auction requires, for firm 1 when it has type $\underline{\beta}$:

$$\underset{\beta^2}{E}\, t_\sigma^1(\underline{\beta},\beta^2)-\underset{\beta^2}{E}\, x_\sigma^1(\underline{\beta},\beta^2)\psi(\underline{\beta}-C_\sigma^1(\underline{\beta},\beta^2))$$

$$\geqq \underset{\beta^2}{E}\, t_\sigma^1(\bar{\beta},\beta^2)-\underset{\beta^2}{E}\, x_\sigma^1(\bar{\beta},\beta^2)\psi(\underline{\beta}-C_\sigma^1(\bar{\beta},\beta^2)). \qquad \text{(IC 1)}$$

Similarly, incentive compatibility for firm 2, when it has type $\underline{\beta}$, requires that:

$$\begin{aligned}&\underset{\beta^1}{E}\, t_\sigma^2(\beta^1,\underline{\beta})-\underset{\beta^1}{E}\, x_\sigma^2(\beta^1,\underline{\beta})\psi(\underline{\beta}-C_\sigma^2(\beta^1,\underline{\beta}))\\&\quad\geqq \underset{\beta^1}{E}\, t_\sigma^2(\beta^1,\bar{\beta})-\underset{\beta^1}{E}\, x_\sigma^2(\beta^1,\bar{\beta})\psi(\underline{\beta}-C_\sigma^2(\beta^1,\bar{\beta})).\end{aligned} \tag{IC 2}$$

Individual rationality for firms 1 and 2 when having type $\bar{\beta}$ requires that:

$$\underset{\beta^2}{E}\, t_\sigma^1(\bar{\beta},\beta^2)-\underset{\beta^2}{E}\, x_\sigma^1(\bar{\beta},\beta^2)\psi(\bar{\beta}-C_\sigma^1(\bar{\beta},\beta^2))\geqq 0, \tag{IR 1}$$

$$\underset{\beta^1}{E}\, t_\sigma^2(\beta^1,\bar{\beta})-\underset{\beta^1}{E}\, x_\sigma^2(\beta^1,\bar{\beta})\psi(\bar{\beta}-C_\sigma^2(\beta^1,\bar{\beta}))\geqq 0. \tag{IR 2}$$

From incentive theory we guess that we can ignore the other incentive and individual rationality constraints and check ex post that they are satisfied by the solution to the subconstrained problem. Since transfers are costly, the above (IC) and (IR) constraints are tight. We can henceforth obtain the rents of asymmetric information which must be given up to the good types:

$$\begin{aligned}U_\sigma^1(\underline{\beta})&=\underset{\beta^2}{E}\, t_\sigma^1(\underline{\beta},\beta^2)-\underset{\beta^2}{E}\, x_\sigma^1(\underline{\beta},\beta^2)\psi(\underline{\beta}-C_\sigma^1(\underline{\beta},\beta^2))\\&=\underset{\beta^2}{E}\, x_\sigma^1(\bar{\beta},\beta^2)\{\psi(\bar{\beta}-C_\sigma^1(\bar{\beta},\beta^2))-\psi(\underline{\beta}-C_\sigma^1(\bar{\beta},\beta^2))\}\\&=\nu x_\sigma^1(\bar{\beta},\underline{\beta})\{\psi(\bar{\beta}-C_\sigma^1(\bar{\beta},\underline{\beta}))-\psi(\underline{\beta}-C_\sigma^1(\bar{\beta},\underline{\beta}))\}\\&\quad+(1-\nu)x_\sigma^1(\bar{\beta},\bar{\beta})\{\psi(\bar{\beta}-C_\sigma^1(\bar{\beta},\bar{\beta}))-\psi(\underline{\beta}-C_\sigma^1(\bar{\beta},\bar{\beta}))\}\\&=\nu x_\sigma^1(\bar{\beta},\underline{\beta})\Phi(e_\sigma^1(\bar{\beta},\underline{\beta}))+(1-\nu)x_\sigma^1(\bar{\beta},\bar{\beta})\Phi(e_\sigma^1(\bar{\beta},\bar{\beta})),\end{aligned}$$

where $e_\sigma^1(\bar{\beta},\underline{\beta})=\bar{\beta}-C_\sigma^1(\bar{\beta},\underline{\beta})$, $e_\sigma^1(\bar{\beta},\bar{\beta})=\bar{\beta}-C_\sigma^1(\bar{\beta},\bar{\beta})$, $\Phi(e)=\psi(e)-\psi(e-\Delta\beta)$. Similarly

$$U_\sigma^2(\underline{\beta})=\nu x_\sigma^2(\underline{\beta},\bar{\beta})\Phi(e_\sigma^2(\underline{\beta},\bar{\beta}))+(1-\nu)x_\sigma^2(\bar{\beta},\bar{\beta})\Phi(e_\sigma^2(\bar{\beta},\bar{\beta})).$$

Let S_σ^i denote the expected valuation of the project done by firm i conditionally on the information σ.

$$S_1^1=\bar{S},\quad S_2^1=\underline{S},\quad S_0^1=(\bar{S}+\underline{S})/2,$$

$$S_1^2=\underline{S},\quad S_2^2=\bar{S},\quad S_0^2=(\bar{S}+\underline{S})/2.$$

In state of information σ, expected social welfare is:

$$\begin{aligned}W_\sigma = \underset{\beta^1,\beta^2}{E}\ &\{S_\sigma^1-(1+\lambda)(\beta^1-e_\sigma^1(\beta^1,\beta^2)+\psi(e_\sigma^1(\beta^1,\beta^2))+s^*)\}\\ &\times x_\sigma^1(\beta^1,\beta^2)-\lambda v U_\sigma^1(\underline{\beta})\\ &+\underset{\beta^1,\beta^2}{E}\ \{S_\sigma^2-(1+\lambda)(\beta^2-e_\sigma^2(\beta^1,\beta^2)+\psi(e_\sigma^2(\beta^1,\beta^2))+s^*)\}\\ &\times(1-x_\sigma^1(\beta^1,\beta^2))-\lambda v U_\sigma^2(\underline{\beta}).\end{aligned}\tag{3.1}$$

Maximizing expected social welfare with respect to $(e_\sigma^i(\cdot),\ x_\sigma^i(\cdot))$ yields:

Proposition 1. When the agency is benevolent, the optimal auction is characterized by:

(1) (i) *If* $\sigma=1$,

$$x_1^1(\bar{\beta},\bar{\beta})=x_1^1(\underline{\beta},\bar{\beta})=x_1^1(\underline{\beta},\underline{\beta})=1,$$

$$\begin{aligned}x_1^1(\bar{\beta},\underline{\beta})=1 \Leftrightarrow \Delta S-(1+\lambda)\,\Delta\beta \geqq (1+\lambda)\\ \times[\psi(\hat{e})-\hat{e}+(\lambda/(1+\lambda))(v/(1-v))\Phi(\hat{e})-\psi(e^*)+e^*]>0.\end{aligned}$$

(ii) *If* $\sigma=2$,

$$x_2^2(\bar{\beta},\bar{\beta})=x_2^2(\bar{\beta},\underline{\beta})=x_2^2(\underline{\beta},\underline{\beta})=1,$$

$$\begin{aligned}x_2^2(\underline{\beta},\bar{\beta})=1 \Leftrightarrow \Delta S-(1+\lambda)\,\Delta\beta \geqq (1+\lambda)\\ \times[\psi(\hat{e})-\hat{e}+(\lambda/(1+\lambda))\,(v/(1-v))\Phi(\hat{e})-\psi(e^*)+e^*]>0.\end{aligned}$$

(iii) *If* $\sigma=0$,

$x_0^1(\bar{\beta},\bar{\beta})$ *and* $x_0^1(\underline{\beta},\underline{\beta})$ *indeterminate in* $[0,1]$,

$x_0^1(\underline{\beta},\bar{\beta})=1$ *and* $x_0^1(\bar{\beta},\underline{\beta})=0$.

(2) *The effort level of a* $\underline{\beta}$ *firm if selected is the efficient level* e^*.
The effort level of a $\bar{\beta}$ *firm if selected is* $\hat{e}$ *defined by*

$$\hat{e} = \underset{e}{\operatorname{argmax}} \, [\psi(e) - e + (\lambda/(1+\lambda))\,(v/(1-v))\,\Phi(e)].$$

Proof. See appendix 1.

The intuition for the optimal auction is clear. If the regulator is informed about quality, the preference is given to the high quality firm when it is at least as efficient as the low quality firm.

When the high quality firm is less efficient than the low quality firm, it is still favored as long as ΔS is larger than

$$(1+\lambda)\,\Delta\beta + (1+\lambda)\left[\left\{\psi(\hat{e}) - \hat{e} + \frac{\lambda}{1+\lambda}\frac{v}{(1-v)}\Phi(\hat{e})\right\} - \{\psi(e^*) - e^*\}\right]. \quad (3.2)$$

The first term is the cost disadvantage already present under complete information and the second is the increase in cost of the less efficient firm due to asymmetric information. Effort is not optimal $\hat{e} \neq e^*$ because using the $\bar{\beta}$ firm increases the rent that must be given up to a $\underline{\beta}$ firm when selected, which has expected social cost $\lambda(v/(1-v))\Phi(\hat{e})$.

Under incomplete information the quality advantage is decisive less often than under complete information. Last, an uninformed agency may use a symmetric auction (choose $x_0^1(\bar{\beta}, \bar{\beta}) = x_0^1(\underline{\beta}, \underline{\beta}) = 1/2$).

Remark 1. We check that the ignored incentive constraints for the $\bar{\beta}$-types are satisfied by the auction defined in Proposition 1. This results directly from the facts that $x_\sigma^1(\underline{\beta}, \beta^2) \geqq x_\sigma^1(\bar{\beta}, \beta^2)$ and $e_\sigma^1(\underline{\beta}, \beta^2) \geqq e_\sigma^1(\bar{\beta}, \beta^2)$ for all β^2 and σ and that the IC constraints for the $\underline{\beta}$ types are binding; and symmetrically for firm 2 (allocations are 'monotonic' in the firm's type).

Remark 2. Firm i's rent is highest for signal $\sigma = i$, as one would expect. It is weakly higher under signal $\sigma = 0$ than under signal $\sigma = j \neq i, 0$.

Remark 3. The dichotomy exhibited in Laffont–Tirole (1987) and McAfee–McMillan (1987) holds also here. The effort levels of the selected firm are identical to those which would be obtained if the regulator was facing a single firm and are defined by e^* for the $\underline{\beta}$ firm and by $\hat{e}$ for a $\bar{\beta}$ firm. Proposition 1 defines the optimal effort levels and the optimal selection variables x. The good type's rent associated with the optimal auction is for firm 1:

$$U_\sigma^1(\underline{\beta}) = v x_\sigma^1(\bar{\beta}, \underline{\beta})\Phi(e_\sigma^1(\bar{\beta}, \underline{\beta})) + (1-v) x_\sigma^1(\bar{\beta}, \bar{\beta})\Phi(e_\sigma^1(\bar{\beta}, \bar{\beta})),$$

and the optimal expected transfers are

$$t_\sigma^1(\underline{\beta}) = U_\sigma^1(\underline{\beta}) + \psi(e^*) \mathop{E}_{\beta^2} x_\sigma^1(\underline{\beta}, \beta^2),$$

$$t_\sigma^1(\bar{\beta}) = \psi(\hat{e}) \mathop{E}_{\beta^2} x_\sigma^1(\bar{\beta}, \beta^2),$$

and similarly for firm 2. The ex post transfers $t_\sigma^1(\beta^1, \beta^2)$ are not determined; only their expectations are.

4. Collusion and soft information

4.1. Description of collusion

From now on we allow the agency to collude with specific bidders. We first introduce the distinction between soft and hard information, distinction that was irrelevant in the absence of collusion. Hard information is information that can be substantiated. That is, the principal can verify the agency's information, if transmitted. The agency's degree of freedom then stems from the possibility of retaining information (reporting $r=0$ when $\sigma=1$ or 2). Formally, $r \in \{\sigma, 0\}$. In contrast, soft information cannot be verified. For any realized signal, the agency can claim to have received any of the possible three signals without being detected. In the case of hard information, we will assume that only the agency can bring evidence about which firm it prefers. For simplicity, we will also assume that even though the firms cannot bring hard evidence on the quality parameter, they learn the signal received by the agency; this assumption limits asymmetries of information in the design of side contracts (see below) and is not crucial; it is easily seen that if the agency wants to collude with a specific firm not to disclose its signal, it is in its interest to show this (hard) information to the firm to convince it of the benefit of colluding.

Next, we assume the following timing: First, the principal publicly offers primary contracts (an auction) to the agency and the firm, which specify the winner and primary transfers to the agency and the firm as functions of all (simultaneous) announcements (report of signal by agency and announcements of technological parameters by the firms) and the winner's realized cost. Second, in sections 4.1, 4.2 and 5, we do not make specific assumptions on the collusion game, i.e., on who makes the offers for side contracts. In sections 4.3 and 6, we specialize the collusion game by assuming that the firms make take-it-or-leave-it offers to the agency for side contracts. A side contract specifies a secondary or side transfer between the two concerned parties that may be contingent on all announcements and the winner's realized cost. Each firm is free to accept or reject its side contract, and its decision is not observed by the other firms. Third, announcements are made, the winner is selected as specified in the auction set up by the principal, and production, primary and side transfers occur. This formulation has the simplicity of avoiding 'signaling phenomena', in particular of not letting the parties with private information (the firms) influence the design of side contracts.

The timing can be summarized as follows:

All parties learn that $\beta \in \{\underline{\beta}, \bar{\beta}\}$ and $\sigma \in \{0, 1, 2\}$. Agency learns σ. Firm i learns β^i and σ.	Principal designs an auction	Side contracting	Announcements	Selection Production	Transfers

We allow side transfers to be costly. An income equivalent of \$1 transferred by firm i to the agency costs $\$(1+\lambda^i)$ of firm i. The parameter $\lambda^i \geqq 0$ is a measure of the deadweight loss of collusive transfers for the two parties [see Laffont–Tirole (1988) for a discussion]. In sections 5 and 6, we will focus on two cases with hard information: *asymmetric collusion*, in which $\lambda^1 \equiv \lambda_f < +\infty$ and $\lambda^2 = +\infty$ (only firm 1 can collude with the agency), and *symmetric collusion*, in which $\lambda^1 = \lambda^2 \equiv \lambda_f$.

4.2. Soft information

The case of soft information has a simple implication in our context. Because quality does not enter the agency's and the firms' objective functions, for a given set of primary contracts (auction), the set of equilibria of the collusion and announcements continuation game is independent of the realisation of the quality signal. We will adhere to the 'Markov principle' that strategically equivalent games or subgames should have the same equilibrium. This principle implies that the same continuation equilibrium prevails for all possible *quality signals received by the agency*,[12] and thus that quality differentials are never decisive; the agency has no discretion in that its announcement has no effect on selection.

This assumption implies that the final allocation is insensitive to the quality signal for a given auction. Therefore the outcome can be implemented without paying attention to the agency's information. We are thus in

[12]In other words, we do not allow the quality signal to play the role of a 'correlating device' or 'sunspot'. For a general definition of Markov perfect equilibrium, see Maskin–Tirole (1989). (The reason why the allocation was responsive to even soft information in section 3 is that the level of quality entered the benevolent regulator's preferences).

the case $\sigma=0$ of section 3, except that we can allow side transfers between the agency and the firms. Because side transfers involve a deadweight loss, all transfers are cheaper to achieve through the principal. We thus conclude that the optimal auction is the symmetric auction corresponding to signal $\sigma=0$. *Quality differentials are never decisive* because no attention is paid to the quality signal; the agency has no discretion in that its announcement has no effect on selection.

Remark. The case of soft information is meant to illustrate some potentially extreme implications of collusion for auction design. It by no means implies that soft information always leads to a rigid auction, in which decentralized information about quality is systematically ignored. In particular, suppose that quality affects the agency's utility as well as the principal's (as is particularly relevant in the example of the European Community). Then even soft information can be used in the presence of collusion. The point is that the agency's report may be made incentive compatible through appropriate transfers because it affects its utility. For a technically similar example in which soft information conditions the equilibrium allocation, see section 7 of Laffont–Tirole (1988).

We observed that if we adhere to the viewpoint that strategically equivalent games or subgames should have the same equilibrium, no use can be made of the agency's signal. This does not imply that the agency has no role because it may perform other tasks than collecting information about quality. This also does not imply that the principal can guarantee himself the collusion - proof payoff for $\sigma=0$: While we deliberately ignored collusion between bidders (bid rigging) to focus on favoritism, bid rigging might still arise in a roundabout way through side contracts between the agency and the firms. For instance, the agency might act as a 'cartel ringmaster' [to use a phrase employed by Krattenmaker and Salop (1986) in a vertical restraints context] and induce each firm to announce 'high' ($\bar{\beta}$) and be rewarded by both. (This is vague as we have not described how this could be implemented through side contracts; this is only meant to illustrate the possibility of indirect bid rigging). We investigate this possibility in the next section.

4.3. Indirect bid rigging

The study of collusion with several informed parties is complex. The outcome depends on the bargaining process for collusion as well as, possibly, on the equilibrium selection. In this section, we derive an upper bound on welfare under collusion and soft information. The upper bound, which turns out to be equal to the one mentioned in the previous paragraph, is obtained when the agency cannot coordinate collusion between the two firms and thus does not act as a cartel ringmaster.

We observed above that the principal can obtain at most W_0. That this upper bound may be reached for some bargaining process for collusive contracts can be seen as follows. Suppose that the firms simultaneously make take-it-or-leave-it offers of side contracts to the agency unlike the case considered in the rest of the paper in which the agency makes two take-it-or-leave-it offers). One can imagine that the firms bargain secretly with the agency and have all the bargaining power. Let the principal offer the symmetric auction for $\sigma=0$ characterized in Proposition 1.

We claim that both firms' offering 'no side contract' (i.e., the side contract that specifies zero side transfers whatever happens) and the two firms' announcing their technological parameters truthfully is an equilibrium. For, suppose that firm i expects firm j and the agency not to enter a side contract and firm j to announce its parameter truthfully. Then from incentive compatibility of the auction, firm i cannot do better than announcing the truth and there is nothing the agency can do to improve its welfare. [In section 6, we will give a more general definition of 'bilaterally interim efficient allocations', which are allocations that are interim efficient from the point of view of a firm and the agency, taking as given the behavior of the other firms. Such allocations cannot give rise to bilateral collusion only, and require multiple collusive arrangements.] Thus an optimal strategy for firm i is to offer no side contract and tell the truth (if firm i could do strictly better than in the allocation of Proposition 1, then by bilateral interim efficiency the agency would lose, which is impossible because it can guarantee itself s^* by not entering side contracts).

What is allowing the upper bound to be reached is clear. Coordination to announce high cost parameters may not be possible if firms offer side contracts.

It is relatively straightforward to derive the principal's welfare when firms collude as if they had complete information about each other. However, we feel that this approach fails to recognize the major difficulty of bargaining under incomplete information. Therefore, we leave open the problem of characterizing the optimal auction when bid rigging is possible.

5. Asymmetric collusion and hard information

Suppose now that the agency's information is hard and that the agency can collude only with firm 1.

We first claim that the principal can obtain the same welfare as under a benevolent agency by adequately picking a variable left indeterminate in Proposition 1. Suppose that the auction is defined as in Proposition 1 except that $x_0^1(\bar{\beta},\bar{\beta})=0$, ($x_0^1(\underline{\beta},\underline{\beta})$ is still indeterminate in [0, 1]). We now show that this auction does not give rise to a side contract between the agency and firm 1 and therefore yields the same welfare to the principal as in the absence of collusion. On the one hand, only type $\underline{\beta}$ of firm 1 may want to bribe the agency to hide its information as type $\bar{\beta}$ gets a zero rent in each state of nature. On the other hand, the type $\underline{\beta}$'s rents, in expectation over firm 2's technological parameter, is the following function of the report r:

$$(1-v)x_r^1(\underline{\beta},\bar{\beta})\Phi(e_r^1(\underline{\beta},\bar{\beta}))+vx_r^1(\underline{\beta},\underline{\beta})\Phi(e_r^1(\underline{\beta},\underline{\beta}))$$

which, for the auction specified by Proposition 1, is equal to:

$$\begin{cases} [(1-v)+vx_1^1(\underline{\beta},\underline{\beta})]\Phi(\hat{e})>0 & \text{if} \quad r=1 \\ 0 & \text{if} \quad r=2 \\ 0 & \text{if} \quad r=0. \end{cases}$$

Because, under hard information, the agency can only hide information away from the principal, firm 1 cannot gain from inducing the agency to retain information (to induce $r=0$). We thus conclude that the auction specified in Proposition 1, with $x_0^1(\bar{\beta},\bar{\beta})=0$ is collusion-proof; and clearly it is optimal in the class of collusion-proof auctions.

Second, we claim that the principal cannot do better with an auction that gives rise to a side contract. The proof of this is very similar to the proof of the collusion-proofness principle for a single firm and hard information in Laffont–Tirole (1988), and is omitted. The reason for this similarity is that firm 2 cannot collude and is therefore much like a dummy firm. Once the incentive cost is included to obtain firm 2's generalized cost, firm 2 can be regarded as a backstop technology. The asymmetric collusion model is really a one-firm model. We thus obtain:

Proposition 2. Suppose that the agency can collude only with firm 1 and that information is hard. The threat of collusion imposes no welfare loss on the principal as long as firm 2 is favored at equal cost when no information about quality is transmitted to the principal.

Remark 1. The main conclusion in Proposition 2 is that firm 2 should be favored when no information about quality is disclosed, in order to induce the agency to reveal information unfavorable to firm 1. The conclusion that asymmetric collusion imposes no welfare loss seems less robust. For, suppose that the β^i are drawn from a continuous distribution. Then, the indeterminacy of $x_0^1(\beta^i,\beta^i)$ under no collusion has probability 0 over the set of (β^i,β^j).

Resolving this indeterminacy in favor of firm 2 apparently does not suffice to yield collusion proofness of the optimal no-collusion auction.[13]

Remark 2. Because the agency can collude with only one firm, we do not need to consider indirect bid rigging.

Remark 3. In the EEC example, the costs envisioned here may be 'generalized costs' if the government attaches some value to the domestic firm's being selected, say for secrecy reasons.

6. Symmetric collusion and hard information

We now allow the agency to collude with the two firms (symmetric collusion). This section is to a large extent exploratory because the development of techniques to study collusion with several informed parties is outside the scope of this paper. We will content ourselves with requiring that the auction offered by the principal (i) induces truthtelling by the three parties in the absence of collusion (ii) be 'bilaterally interim efficient'. We will say that an allocation is bilaterally interim efficient if there exists no vector of side-transfers between the agency and a firm i and no announcement strategy by the agency and this firm that is incentive compatible given the original auction and the side transfers that yields a Pareto superior allocation for the agency and firm i, *taking firm j's announcement strategy (i.e., telling the truth) as given.*

We do not offer a complete defense of this requirement, but we make the following points. Assume that the extensive form for the collusion game has the firms make take-or-leave-it offers of side contracts to the agency (and that these offers are secret) and suppose that the principal offers a bilaterally interim efficient allocation. Then the absence of collusion (each firm's offering the null contract) followed by truthtelling is an equilibrium: knowing that the other firm does not offer a side contract and subsequently tells the truth, each firm has no incentive to offer a side contract, because by bilateral interim efficiency it either loses expected utility or the agency loses expected utility in which case the agency turns the side contract down.

In our context, bilteral interim efficiency is equivalent to imposing the extra requirement that no firm has an incentive to bribe the agency to hide its information [conditions (6.1) and (6.2) below]. If either condition is violated, then the firms' offering the null side contract and truthtelling by all parties is not an equilibrium.

To obtain bilateral interim efficiency, it must be the case that if firm i has

[13] It is worth mentioning why we chose to work with a two-type space. With two types, collusive activities necessarily stem from type $\underline{\beta}$ because type $\bar{\beta}$ gets no rent. With more than two types, the agency must screen in a more subtle way the firm's willingness to pay for collusion.

cost parameter $\underline{\beta}$ (and therefore enjoys a rent) and the agency receives the signal that firm $j \neq i$ offers a higher quality, firm i has no incentive to induce the agency to retain its information. Let s_j $(j=1,2)$ denote the agency's income when it reports $r=j$ and firm $i \neq j$ announces $\hat{\beta}^i = \underline{\beta}$. As is easily seen the other contingent incomes for the agency are optimally set at s^* as the threat of collusion operates only in the above case.

Let

$$A_1 \equiv s_1 - s^* - (1/(1+\lambda_f))\,[(1-\nu)x_0^2(\bar{\beta},\bar{\beta})\Phi(e_0^2(\bar{\beta},\bar{\beta}))$$

$$+\nu x_0^2(\underline{\beta},\bar{\beta})\Phi(e_0^2(\underline{\beta},\bar{\beta})) - (1-\nu)x_1^2(\bar{\beta},\bar{\beta})\Phi(e_1^2(\bar{\beta},\bar{\beta}))$$

$$-\nu x_1^2(\underline{\beta},\bar{\beta})\Phi(e_1^2(\underline{\beta},\bar{\beta}))]. \tag{6.1}$$

$$A_2 \equiv s_2 - s^* - (1/(1+\lambda_f))\,[(1-\nu)x_0^1(\bar{\beta},\bar{\beta})\Phi(e_0^1(\bar{\beta},\bar{\beta}))$$

$$+\nu x_0^1(\bar{\beta},\underline{\beta})\Phi(e_0^1(\bar{\beta},\underline{\beta})) - (1-\nu)x_2^1(\bar{\beta},\bar{\beta})\Phi(e_2^1(\bar{\beta},\bar{\beta}))$$

$$-\nu x_2^1(\bar{\beta},\underline{\beta})\Phi(e_2^1(\bar{\beta},\underline{\beta}))]. \tag{6.2}$$

Bilateral interim efficiency is equivalent to $A_1 \geqq 0$, $A_2 \geqq 0$. Indeed, the only case when collusion between the agency and firm 2 is valuable is $\sigma=1$, $\beta_2=\underline{\beta}$. Then, if the agency withholds its information $(r=0)$, firm 2 of type $\underline{\beta}$ may obtain a rent. $A_1 \geqq 0$ says that, for $r=1$ to $r=0$, the expected rent increase of firm 2 (when it is of type $\underline{\beta}$ and claims that it is of type $\bar{\beta}$), appropriately discounted to take into account that internal transfers within the coalition are costly, is inferior to the loss that the agency would incur from such an untruthful report. Colluding and claiming $\beta=\bar{\beta}$ would not be more valuable since, as incentive constraints are binding, a type $\underline{\beta}$ firm is indifferent between announcing $\beta=\underline{\beta}$ or $\beta=\bar{\beta}$. Similarly for $A_2 \geqq 0$.

The principal wishes to maximize:

$$\xi W_1 + \xi W_2 + (1-2\xi)W_0 - \lambda\nu\xi(s_1 - s^*) - \lambda\nu\xi(s_2 - s^*) \tag{6.3}$$

under the constraints

$$A_1 \geqq 0;\ A_2 \geqq 0;\ s_1 \geqq s^*;\ s_2 \geqq s^*,$$

where W_σ is defined in equation (3.1).

Lemma 1. At the optimum of program (6.3) $A_1=0$ *and* $A_2=0$.

Proof. Suppose that $A_1 > 0$. Then the shadow price of constraint (6.1) is equal to zero and $s_1 = s^*$. The maximization is then the same as that in section 5 where only firm 1 can collude. We know that the solution then involves $x_0^2(\bar{\beta}, \bar{\beta}) = 0$ and $s_2 = s^*$. Then $A_2 = 0$ but $A_1 < 0$ a contradiction. And similarly if $A_2 > 0$. *Q.E.D.*

We now give a full description of the optimal bilaterally interim efficient auction, and later interpret its findings.

Let $\hat{\hat{e}}$ and $\breve{e}(e^* > \hat{\hat{e}} > \hat{e} > \breve{e})$ be defined by

$$\hat{\hat{e}} = \operatorname*{argmax}_e \left\{ \psi(e) - e + \frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu}\,\frac{\lambda_f}{1+\lambda_f}\,\Phi(e) \right\},$$

$$\breve{e} = \operatorname*{argmax}_e \left\{ \psi(e) - e + \frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu}\left(1 + \frac{\xi}{(1-2\xi)(1+\lambda_f)}\right)\Phi(e) \right\}.$$

Let $\{\hat{e}(\Delta S), \mu(\Delta S)\}$ the solution $\{e, \mu\}$ of:

$$\Delta S = (1+\lambda)\left[\left\{ \psi(\hat{\hat{e}}) - \hat{\hat{e}} + \frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu}\,\Phi(\hat{\hat{e}}) \right\} \right.$$

$$\left. - \left\{ \psi(e) - e + \left(\frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu} - \frac{\mu}{(1-2\xi)(1-\nu)(1+\lambda)} \right)\Phi(e) \right\} \right],$$

and

$$\psi'(e) = 1 - \left(\frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu} - \frac{\mu}{(1-2\xi)(1-\nu)(1+\lambda)} \right)\Phi'(e).$$

Let $\breve{e}(\Delta S)$ defined by

$$\psi'(\breve{e}(\Delta S)) = 1 - \left(\frac{\lambda}{1+\lambda} \cdot \frac{\nu}{1-\nu} + \frac{\mu(\Delta S)}{(1-2\xi)(1-\nu)(1+\lambda)} \right)\Phi'(\breve{e}(\Delta S)).$$

Proposition 3. The solutions to program (6.3) are characterized by:

Case 1. $\Delta S > (1+\lambda)\left[\left\{ \psi(\hat{\hat{e}}) - \hat{\hat{e}} + \frac{\lambda}{1+\lambda} \cdot \frac{\nu}{1-\nu}\,\Phi(\hat{\hat{e}}) \right\} \right.$

$$-\left\{\psi(\hat{\hat{e}})-\hat{\hat{e}}+\frac{\lambda}{1+\lambda}\cdot\frac{v}{1-v}\frac{\lambda_f}{1+\lambda_f}\Phi(\hat{\hat{e}})\right\}\Bigg].$$

If $\sigma=1$:

$$x_1^1(\bar{\beta},\bar{\beta})=x_1^1(\underline{\beta},\underline{\beta})=x_1^1(\underline{\beta},\bar{\beta})=1,$$

$$x_1^1(\bar{\beta},\underline{\beta})=1\Leftrightarrow\Delta S-(1+\lambda)\,\Delta\beta>(1+\lambda)$$

$$\times\left[\left\{\psi(\hat{e})-\hat{e}+\frac{\lambda}{1+\lambda}\cdot\frac{v}{1-v}\Phi(\hat{e})-\{\psi(e^*)-e^*\}\right\}\right],$$

$$e_1^1(\bar{\beta},\bar{\beta})=e_1^1(\bar{\beta},\underline{\beta})=\hat{e},\qquad e_1^2(\underline{\beta},\bar{\beta})=\hat{\hat{e}},$$

$$e_1^1(\underline{\beta},\bar{\beta})=e_1^1(\underline{\beta},\underline{\beta})=e_1^2(\underline{\beta},\bar{\beta})=e^*.$$

If $\sigma=2$, *the solution is symmetric.*
If $\sigma=0$:

$$x_0^1(\bar{\beta},\underline{\beta})=x_0^2(\underline{\beta},\bar{\beta})=0,$$

$$x_0^1(\underline{\beta},\underline{\beta})\ \textit{and}\ x_0^1(\bar{\beta},\bar{\beta})\in[0,1],$$

$$e_0^1(\underline{\beta},\bar{\beta})=e_0^2(\bar{\beta},\underline{\beta})=e_0^1(\underline{\beta},\underline{\beta})=e_0^2(\underline{\beta},\underline{\beta})=e^*,$$

$$e_0^1(\bar{\beta},\underline{\beta})=e_0^2(\underline{\beta},\bar{\beta})=e_0^1(\bar{\beta},\bar{\beta})=e_0^2(\bar{\beta},\bar{\beta})=\breve{e},$$

$$s_1=s^*+\frac{1-v}{1+\lambda_f}\,x_0^2(\bar{\beta},\bar{\beta})\Phi(\breve{e}),$$

$$s_2=s^*+\frac{1-v}{1+\lambda_f}\,x_0^1(\bar{\beta},\bar{\beta})\Phi(\breve{e}).$$

Case 2. $\Delta S<(1+\lambda)\Bigg[\left\{\psi(\hat{e})-\hat{e}+\frac{\lambda}{1+\lambda}\cdot\frac{v}{1-v}\Phi(\hat{e})\right\}$

$$-\left\{\psi(\hat{\hat{e}})-\hat{\hat{e}}+\frac{\lambda}{1+\lambda}\cdot\frac{v}{1-v}\frac{\lambda_f}{1+\lambda_f}\Phi(\hat{\hat{e}})\right\}\Bigg].$$

$s_1=s_2=s^*.$

The solution is as in Case 1 except that

$\hat{e}$ *is replaced by* $\hat{e}(\Delta S)$,

$\check{e}$ *is replaced by* $\check{e}(\Delta S)$,

$$x_1^2(\bar{\beta},\bar{\beta})=\frac{\Phi(\check{e}(\Delta S))}{\Phi(\hat{e}(\Delta S))}\cdot x_0^2(\bar{\beta},\bar{\beta}),$$

$$x_2^1(\bar{\beta},\bar{\beta})=\frac{\Phi(\check{e}(\Delta S))}{\Phi(\hat{e}(\Delta S))}\cdot x_0^1(\bar{\beta},\bar{\beta}),$$

with $x_0^2(\bar{\beta},\bar{\beta})+x_0^1(\bar{\beta},\bar{\beta})=1.$

A symmetric solution is obtained with $x_0^2(\bar{\beta},\bar{\beta})=x_0^1(\bar{\beta},\bar{\beta})=1/2.$

Proof. See appendix 2.

Interpretation: Two different ways of satisfying the collusion constraints are described in cases 1 and 2.

In case 1, the constraint is satisfied by motivating the agency with appropriate transfers. This is the case where the agency's information is valuable (ΔS large) and therefore worth obtaining. A necessary condition for case 1 to obtain is that quality differentials be decisive in the absence of collusion. The allocation (selection rule, effort) is the same as is the absence of collusion when $\sigma=1$ or 2, but incentives are lowered when $\sigma=0$.

In case 2 (low ΔS) the agency is not motivated but the stakes of collusion are nullified by making the auction closer to a symmetric auction when $(\beta_1,\beta_2)=(\bar{\beta},\bar{\beta})$ and by decreasing the effort levels of the bad types (and consequently lowering the power of the incentive schemes). If we choose $x_0^1(\bar{\beta},\bar{\beta})=x_0^2(\bar{\beta},\bar{\beta})=1/2$, as $\Delta S\to 0$, $\mu\to 0$, $\check{e}(\Delta S)\to\hat{e}$, $\hat{e}\to\hat{e}$ and $x_1^2(\bar{\beta},\bar{\beta})=x_2^1(\bar{\beta},\bar{\beta})\to 1/2$. We obtain a *strictly symmetric auction in the limit when quality differentials become small.*

Note that when the costs of collusion (λ_f) increase we are more likely to be in case 1 where quality differentials matter in awarding a contract because the agency is motivated to be truthful.

Last, to completely prevent collusion it must be the case that the agency when it has signal $\sigma=i$ has no incentive to bribe the $\bar{\beta}$ firm j to claim $\hat{\beta}^j=\underline{\beta}$. In case 2 this condition is automatically satisfied since the agency's income is always s^*. In case 1 let $1+\lambda_a$ the cost of transfers from the agency to a firm. The no collusion constraint when $\sigma=1$ is that the agency does not want to offer more than the loss incurred by firm 2, i.e.

$$\frac{1}{1+\lambda_a}\frac{(1-v)^2}{1+\lambda_f}\frac{1}{2}\Phi(\check{e})\leqq 0$$

if $\Delta S-(1+\lambda)\Delta\beta>(1+\lambda)$

$$\times\left[\left\{\psi(\hat{e})-\hat{e}+\frac{\lambda}{(1+\lambda)}\frac{v}{(1-v)}\Phi(\hat{e})\right\}-\{\psi(e^*)-e^*\}\right].$$

The right hand side is zero since firm 2 does not produce in that case whatever its β. So $\lambda_a=\infty$ is needed to prevent collusion. If λ_a is not infinite, then the policy described in case 1 of Proposition 3 must be altered. For instance, if λ_a is large, but finite, one can bring $x_1^1(\bar{\beta},\underline{\beta})$ a bit below 1, so that the expected cost for type $\bar{\beta}$ of firm 2 be strictly positive. Thus the conclusions of Proposition 3 remain approximately valid if λ_a is large, but finite.

$$\frac{1}{1+\lambda_a}\frac{(1-v)^2}{1+\lambda_f}\frac{1}{2}\Phi(\check{e})<(1-v)\Phi(e^*+\Delta\beta)$$

if $\Delta S-(1+\lambda)\Delta\beta<(1+\lambda)$

$$\times\left[\left\{\psi(\hat{e})-\hat{e}+\frac{\lambda}{(1+\lambda)}\frac{v}{(1-v)}\Phi(\hat{e})\right\}-\{\psi(e^*)-e^*\}\right].$$

If λ_a (or $\Delta\beta$) is large enough this condition and a symmetric condition when $\sigma=2$ obtain. Otherwise, the auction must be altered by decreasing the transfers to the agency and modifying appropriately effort levels to satisfy all collusion constraints. (The right-hand sides of these equations reflects the fact that the transfer must be made indiscriminately to types $\underline{\beta}$ and $\bar{\beta}$ of firm 2 even though the agency tries to influence only type $\bar{\beta}$'s report.)

7. Conclusion

We first summarize the main implications of our analysis and state some caveats. We then discuss instruments to fight favoritism that were ignored in the model.

Bidders' private information generates rents that are sensitive to the nature of the auction. Bidders suffer from being discriminated against because a lower probability of winning reduces their expected informational rent; their interest thus lies in being favored by the agency. Our analysis predicts that the threat of collusion between the agency and specific bidders tends to reduce the former's discretion in devising an optimal bidding rule. First, acquisition procedures may impose rules on the agency: obligation to widely publicize the auction to reach all potential bidders, to clearly define the object for bid and to publicly disclose actual bids to allow the principal to control the selection process. Second, and more specifically the focus of our paper, the bidding game is modified by the possibility of collusion. In extreme cases (see section 4), the principal forces the agency to set up a symmetric auction even if the latter has information that would warrant discrimination. For instance, if the winner's ex post cost is unobservable so that only a fixed-price contract can be signed,[14] the contract is awarded to the lowest bidder in spite of possible differences in quality among bidders (this procedure corresponds to the 'marchés par adjudication' in France).

Or the principal may leave some discretion to the agency but require it to supply substantiated evidence to vindicate discriminatory decisions. In this respect the procedure differs from the French 'marchés sur appel d'offre' in which the selection committee picks the bidder it prefers and is not required to explain its choice. It is more akin to the U.S. Air Force acquisition procedures in which the source selection authorities must produce ratings by the (in principle separate) source selection evaluation board on factors such as price, reliability of firms or technical merit of the projects.[15] Similarly, since 1988, the European Commission requires governments to provide evidence in support of the use of restricted auctions; it also requires disclosure of information so that firms which feel unfairly discriminated against can appeal.

When the agency can collude only with one bidder, the issue is to encourage him to disclose information that is favorable to rival bidder(s). To this purpose, it is optimal to favor the rival bidder (choose him when costs are roughly the same) when no evidence is provided. Asymmetric possibilities of collusion may thus move optimal auctions away from symmetric auctions. Next, if the agency provides evidence in favor of the colluding bidder, and if

[14]A fixed-price contract makes the winning firm the residual claimant for its cost savings. Our model considers the more general case in which the winner's cost is observable. The case in which this cost is unobservable corresponds to a linear specification of the disutility of effort function: $\psi(e)=e$. Negative effort is then equivalent to theft and cost reimbursement is undesirable. The reader will check that effort is always given by $\psi'(e)=1$.

[15]In this respect, it is interesting to note that a DOD contracting officer who does not select a lowest bidder is supposed to write up a comprehensive justification and defend it and be prepared to face a protest. While such procedures impose lots of extra-work and potential delays, they may be welfare enhancing as suggested by our paper. It is also worth noting that the reduction in discretion of the auction designer emphasized by our theory has its counterpart in defense contracting. It is often felt [see Fox (1974, chapter 13)] that the General Accounting Office and Congress looking over the shoulder of the project managers cause them to do what is apparently safe: make awards on the basis of objective variables (lowest cost estimate, shortest schedule, ...) rather than on subjective ones.

the quality differential is big enough, the agency is allowed to use a restricted 'auction' with only this bidder.

When the principal can collude with any of the bidders equally well, the threat of collusion moves the auction toward a symmetric auction. Quality differentials are less likely to be decisive than in the absence of collusive threat.

To conclude, we would like to discuss some limitations of these results and to mention some alternative instruments to fight favoritism.

First, we assumed that the principal costlessly organizes the auction and the agency contents itself with announcing its information about project quality. In practice, the principal often does not have the resources to organize each and every auction. Rather, like in the case of the European Community, it may give directives on how to design auctions and rely on agents to complain about abuses. In such cases, it exerts ex post rather than ex ante control. This raises the question of whether the appeal procedure is costless for the firms that are unfairly discriminated against.[16] Sometimes, such firms refrain from complaining because they are afraid of being unfairly discriminated against in the future. Further analysis is required to describe the mechanism by which the long-term benefit from having a reputation for not complaining may outweigh the short-term gain from obtaining compensatory damages. But we should note that the European Community is considering making the grievance procedure anonymous. It of course remains to be seen how anonymity can be made compatible with efficient fact finding.

Second, in some industries, the enforcement of fairness rules faces the same problem as the enforcement of the prohibition of some vertical restraints. The buyer may integrate vertically in order to withdraw transactions from the legal realm. This may be a problem when the buyer is not legally an agent for the principal (as in the case when the principal is a legislative or a legal body), and when the buyer is a producer itself, so that the principal cannot prevent vertical integration.

This paper has focused on how auctions of incentive contracts are distorted to thwart favoritism and took the collusion technology as given. There exist complementary methods of fighting favoritism that raise the cost of collusion (λ_f in our model). On the one hand, the principal may put

[16]Marshall–Meurer–Richard (1989) analyze the role of an appeal procedure in defense contracting. They argue that successful protects reduce the return to lobbying, thereby diminishing the incentive to invest in it; and that, because protests are invoked by a firm that uses its superior information, they may be a more appealing device for regulating procurement officials than auditing.

In 1984 Congress passed the Competition in Contracting Act that offers firms the opportunity to protest to the General Accounting office in a quasi-judicial hearing (Marshall) and note that there have been over 3,000 protests per year and that many protestors have received large settlement awards from the winning bidders in exchange for a promise to drop their protest.

restrictions on the interface between auction designer and bidders.[17] And he may (and usually does) select agencies that do not exhibit conflicts of interest. On the other hand, he may divide tasks in the selection process so as to reduce the possibility of collusion. For instance, the theoretical division of labor in the U.S. Air Force acquisition procedures is as follows: the teams of source selection evaluation board rate the various components of bids. The source selection authority, who has solicited proposals, selects the winner. And the source selection advisory council checks that competition has been obtained in the selection process, and reviews and approves evaluation standards. The limits of the division of labor are obvious: it is costly to employ several bodies with high technological competence in the same area; and it must be the case that these bodies do not collude among themselves. But to the extent that they can be kept reasonably independent, the division of labor may reduce collusion.[18]

Last, when the agency handles many independent auctions and can collude with only one category of bidders, the principal can use the 'law of large numbers' to detect collusion. It is interesting in this respect to note that the 1976 directive of the Commission of the European Community requires each country to publish the percentage (in numbers and value) of contracts going to domestic firms.

Appendix 1

Proof of Proposition 1

When $\sigma = 1$, expected social welfare is:

$$v^2[\bar{S} - (1+\lambda)(\underline{\beta} - e_1^1(\underline{\beta},\underline{\beta}) + \psi(e_1^1(\underline{\beta},\underline{\beta}))]x_1^1(\underline{\beta},\underline{\beta})$$

$$+ v^2[\underline{S} - (1+\lambda)(\underline{\beta} - e_1^2(\underline{\beta},\underline{\beta}) + \psi(e_1^2(\underline{\beta},\underline{\beta}))](1 - x_1^1(\underline{\beta},\underline{\beta}))$$

$$+ v(1-v)[\bar{S} - (1+\lambda)(\underline{\beta} - e_1^1(\underline{\beta},\bar{\beta}) + \psi(e_1^1(\underline{\beta},\bar{\beta}))]x_1^1(\underline{\beta},\bar{\beta}))$$

$$+ v(1-v)[\underline{S} - (1+\lambda)(\bar{\beta} - e_1^2(\underline{\beta},\bar{\beta}) + \psi(e_1^2(\underline{\beta},\bar{\beta}))](1 - x_1^1(\underline{\beta},\bar{\beta}))$$

$$+ v(1-v)[\bar{S} - (1+\lambda)(\bar{\beta} - e_1^1(\bar{\beta},\underline{\beta}) + \psi(e_1^1(\bar{\beta},\underline{\beta}))]x_1^1(\bar{\beta},\underline{\beta})$$

[17]According to the *U.S. Air Force acquisition procedure 71-15* (pp. 8–9), 'the objectivity of the source selection process may be impaired by contacts between prospective contractors related to acquisitions in source selection and senior Department personnel during the period between the release of solicitation and announcement of source selection decision. Contacts with prospective contractors must be avoided except for personnel directly responsible for participating in the contract negotiations'.

[18]Similarly in Japan a body different from the auction designer ranks firms in categories A, B, C which define the types of auctions in which they can participate.

$$+\nu(1-\nu)[\underline{S}-(1+\lambda)(\underline{\beta}-e_1^2(\bar{\beta},\underline{\beta})+\psi(e_1^2(\bar{\beta},\underline{\beta}))](1-x_1^1(\bar{\beta},\underline{\beta}))$$

$$+(1-\nu)^2[\bar{S}-(1+\lambda)(\bar{\beta}-e_1^1(\bar{\beta},\bar{\beta})+\psi(e_1^1(\bar{\beta},\bar{\beta}))]x_1^1(\bar{\beta},\bar{\beta})$$

$$+(1-\nu)^2[\underline{S}-(1+\lambda)(\bar{\beta}-e_1^2(\bar{\beta},\bar{\beta})+\psi(e_1^2(\bar{\beta},\bar{\beta}))](1-x_1^1(\bar{\beta},\bar{\beta}))$$

$$-\lambda\nu[\nu x_1^1(\bar{\beta},\underline{\beta})\Phi(e_1^1(\bar{\beta},\underline{\beta}))+(1-\nu)x_1^1(\bar{\beta},\bar{\beta})\Phi(e_1^1(\bar{\beta},\bar{\beta}))]$$

$$-\lambda\nu[\nu(1-x_1^1(\underline{\beta},\bar{\beta}))\Phi(e_1^2(\underline{\beta},\bar{\beta}))+(1-\nu)(1-x_1^1(\bar{\beta},\bar{\beta}))\Phi(e_1^2(\bar{\beta},\bar{\beta})].$$

$$\text{(A.1.1)}$$

Rewriting we get:

$$\nu^2[\bar{S}-(1+\lambda)(\underline{\beta}-e_1^1(\underline{\beta},\underline{\beta})+\psi(e_1^1(\underline{\beta},\underline{\beta}))]x_1^1(\underline{\beta},\underline{\beta}))$$

$$+\nu^2[\underline{S}-(1+\lambda)(\underline{\beta}-e_1^2(\underline{\beta},\underline{\beta})+\psi(e_1^2(\underline{\beta},\underline{\beta}))](1-x_1^1(\underline{\beta},\underline{\beta}))$$

$$+\nu(1-\nu)[\bar{S}-(1+\lambda)(\underline{\beta}-e_1^1(\underline{\beta},\bar{\beta})+\psi(e_1^1(\underline{\beta},\bar{\beta}))]x_1^1(\underline{\beta},\bar{\beta})$$

$$+\nu(1-\nu)\Bigg[\underline{S}-(1+\lambda)(\bar{\beta}-e_1^2(\underline{\beta},\bar{\beta})+\psi(e_1^2(\underline{\beta},\bar{\beta}))$$

$$-\frac{\lambda\nu}{(1-\nu)}\Phi(e_1^2(\underline{\beta},\bar{\beta}))\Bigg](1-x_1^1(\underline{\beta},\bar{\beta}))$$

$$+\nu(1-\nu)\Bigg[\bar{S}-(1+\lambda)(\bar{\beta}-e_1^1(\bar{\beta},\underline{\beta})+\psi(e_1^1(\bar{\beta},\underline{\beta}))$$

$$-\frac{\lambda\nu}{(1-\nu)}\Phi(e_1^1(\bar{\beta},\underline{\beta}))\Bigg]x_1^1(\bar{\beta},\underline{\beta})$$

$$+\nu(1-\nu)[\underline{S}-(1+\lambda)(\underline{\beta}-e_1^2(\bar{\beta},\underline{\beta})+\psi(e_1^2(\bar{\beta},\underline{\beta}))](1-x_1^1(\bar{\beta},\underline{\beta}))$$

$$+(1-\nu)^2\Bigg[\bar{S}-(1+\lambda)(\bar{\beta}-e_1^1(\bar{\beta},\bar{\beta})+\psi(e_1^1(\bar{\beta},\bar{\beta}))$$

$$-\frac{\lambda\nu}{(1-\nu)}\Phi(e_1^1(\bar{\beta},\bar{\beta}))\Bigg]x_1^1(\bar{\beta},\bar{\beta})$$

$$+(1-v)^2\left[\underline{S}-(1+\lambda)(\bar{\beta}-e_1^2(\bar{\beta},\bar{\beta})+\psi(e_1^2(\bar{\beta},\bar{\beta}))\right.$$

$$\left.-\frac{\lambda v}{(1-v)}\Phi(e_1^2(\bar{\beta},\bar{\beta}))\right](1-x_1^1(\bar{\beta},\bar{\beta})). \tag{A.1.2}$$

As the x_1^i are between 0 and 1, the maximization of this expression requires the maximization of each term between brackets and then the choice of $x_1^1=1$ or $x_1^2=1$ according to the magnitude of the terms between brackets.

Take the first two terms. Maximization with respect to effort leads to

$$e_1^1(\underline{\beta},\underline{\beta})=e^*, \qquad e_1^2(\underline{\beta},\underline{\beta})=e^*.$$

As $\bar{S}>\underline{S}$, we must choose $x_1^1(\underline{\beta},\underline{\beta})=1$.

Take then the next two terms. We get

$$e_1^1(\underline{\beta},\bar{\beta})=e^*,$$

$$e_1^2(\underline{\beta},\bar{\beta})=\hat{e}<e^*,$$

where $\hat{e}$ is the solution of

$$\psi'(e)=1-\frac{\lambda}{1+\lambda}\frac{v}{1-v}\Phi'(e),$$

and as $\bar{S}>\underline{S}$, clearly $x_1^1(\underline{\beta},\bar{\beta})=1$.

Taking the last two terms we get similarly

$$e_1^1(\bar{\beta},\bar{\beta})=e_1^2(\bar{\beta},\bar{\beta})=\hat{e}, \qquad \text{and}$$

$$x_1^1(\bar{\beta},\bar{\beta})=1.$$

The interesting piece is composed of the 5th and 6th terms where we get

$$e_1^1(\bar{\beta},\underline{\beta})=\hat{e}$$

$$e_1^2(\bar{\beta},\underline{\beta})=e^*, \qquad \text{and}$$

$$x_1^1(\bar\beta,\underline\beta)=1 \Leftrightarrow \Delta S-(1+\lambda)\,\Delta\beta \geqq (1+\lambda)$$

$$\times\left[\psi(\hat e)-\hat e+\frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu}\,\Phi(\hat e)-(\psi(e^*)-e^*)\right].$$

Appendix 2

Proof of Proposition 3

Suppose first that the constraints $s_1 \geqq s^*$ and $s_2 \geqq s^*$ are not binding. As $A_1=A_2=0$ from Lemma 1, s_1 and s_2 can be substituted into the objective function. The maximization problem becomes:

$$\operatorname{Max}\xi\Big[\nu^2(\bar S-(1+\lambda)(\underline\beta-e_1^1(\underline\beta,\underline\beta)+\psi(e_1^1(\underline\beta,\underline\beta)))x_1^1(\underline\beta,\underline\beta)$$

$$+\nu^2(\underline S-(1+\lambda)(\underline\beta-e_1^2(\underline\beta,\underline\beta)+\psi(e_1^2(\underline\beta,\underline\beta)))(1-x_1^1(\underline\beta,\underline\beta))$$

$$+\nu(1-\nu)\Big(\bar S-(1+\lambda)(\bar\beta-e_1^1(\bar\beta,\underline\beta)+\psi(e_1^1(\bar\beta,\underline\beta)$$

$$+\frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu}\,\Phi(e_1^1(\bar\beta,\underline\beta))\Big)x_1^1(\bar\beta,\underline\beta)$$

$$+\nu(1-\nu)(\underline S-(1+\lambda)(\underline\beta-e_1^2(\bar\beta,\underline\beta)+\psi(e_1^2(\bar\beta,\underline\beta)))(1-x_1^1(\bar\beta,\underline\beta))$$

$$+\nu(1-\nu)(\bar S-(1+\lambda)(\underline\beta-e_1^1(\underline\beta,\bar\beta)+\psi(e_1^1(\underline\beta,\bar\beta))x_1^1(\underline\beta,\bar\beta)$$

$$+\nu(1-\nu)(\underline S-(1+\lambda)(\bar\beta-e_1^2(\underline\beta,\bar\beta)+\psi(e_1^2(\underline\beta,\bar\beta))$$

$$+\frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu}\,\frac{\lambda_f}{(1+\lambda_f)}\,\Phi(e_1^2(\underline\beta,\bar\beta)))x_1^2(\underline\beta,\bar\beta)$$

$$+(1-\nu)^2(\bar S-(1+\lambda)(\bar\beta-e_1^1(\bar\beta,\bar\beta)+\psi(e_1^1(\bar\beta,\bar\beta))$$

$$+\frac{\lambda}{1+\lambda}\,\frac{\nu}{1-\nu}\,\Phi(e_1^1(\bar\beta,\bar\beta)))x_1^1(\bar\beta,\bar\beta)$$

$$+(1-\nu)^2(\underline S-(1+\lambda)(\bar\beta-e_1^2(\bar\beta,\bar\beta)+\psi(e_1^2(\bar\beta,\bar\beta))$$

$$+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\frac{\lambda_f}{(1+\lambda_f)}\Phi(e_1^2(\bar{\beta},\bar{\beta}))x_1^2(\bar{\beta},\bar{\beta})\Bigg]$$

$$+\xi\Bigg[v^2(\bar{S}-(1+\lambda)(\underline{\beta}-e_2^2(\underline{\beta},\underline{\beta})+\psi(e_2^2(\underline{\beta},\underline{\beta}))x_2^2(\underline{\beta},\underline{\beta})$$

$$+v^2(\underline{S}-(1+\lambda)(\underline{\beta}-e_2^1(\underline{\beta},\underline{\beta})+\psi(e_2^1(\underline{\beta},\underline{\beta})))(1-x_2^2(\underline{\beta},\underline{\beta}))$$

$$+v(1-v)(\bar{S}-(1+\lambda)(\underline{\beta}-e_2^2(\bar{\beta},\underline{\beta})+\psi(e_2^2(\bar{\beta},\underline{\beta})))x_2^2(\bar{\beta},\underline{\beta}))$$

$$+v(1-v)(\underline{S}-(1+\lambda)(\bar{\beta}-e_2^1(\bar{\beta},\underline{\beta})+\psi(e_2^1(\bar{\beta},\underline{\beta}))$$

$$+\frac{\lambda}{1+\lambda}\frac{v}{(1-v)}\frac{\lambda_f}{1+\lambda_f}\Phi(e_2^1(\bar{\beta},\underline{\beta}))(1-x_2^2(\bar{\beta},\underline{\beta}))$$

$$+v(1-v)(\bar{S}-(1+\lambda)(\bar{\beta}-e_2^2(\underline{\beta},\bar{\beta})+\psi(e_2^2(\underline{\beta},\bar{\beta}))$$

$$+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\Phi(e_2^2(\underline{\beta},\bar{\beta}))x_2^2(\underline{\beta},\bar{\beta})$$

$$+v(1-v)(\underline{S}-(1+\lambda)(\underline{\beta}-e_2^1(\underline{\beta},\bar{\beta})+\psi(e_2^1(\underline{\beta},\bar{\beta})))(1-x_2^2(\underline{\beta},\bar{\beta}))$$

$$+(1-v)^2(\bar{S}-(1+\lambda)(\bar{\beta}-e_2^2(\bar{\beta},\bar{\beta})+\psi(e_2^2(\bar{\beta},\bar{\beta}))$$

$$+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\Phi(e_2^2(\bar{\beta},\bar{\beta})))x_2^2(\bar{\beta},\bar{\beta})$$

$$+(1-v)^2(\underline{S}-(1+\lambda)(\bar{\beta}-e_2^1(\bar{\beta},\bar{\beta})+\psi(e_2^1(\bar{\beta},\bar{\beta}))$$

$$+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\frac{\lambda_f}{1+\lambda_f}\Phi(e_2^1(\bar{\beta},\bar{\beta})))(1-x_2^2(\bar{\beta},\bar{\beta}))\Bigg]$$

$$+(1-2\xi)\Bigg[v^2\left(\frac{\bar{S}+\underline{S}}{2}-(1+\lambda)(\underline{\beta}-e_0^1(\underline{\beta},\underline{\beta})+\psi(e_0^1(\underline{\beta},\underline{\beta})))\right)x_0^1(\underline{\beta},\underline{\beta})$$

$$+v^2\left(\frac{\bar{S}+\underline{S}}{2}-(1+\lambda)(\underline{\beta}-e_0^2(\underline{\beta},\underline{\beta})+\psi(e_0^2(\underline{\beta},\underline{\beta}))\right)(1-x_0^1(\underline{\beta},\underline{\beta}))$$

$$+v(1-v)\left(\frac{\bar{S}+\underline{S}}{2}-(1+\lambda)(\bar{\beta}-e_0^1(\bar{\beta},\underline{\beta})+\psi(e_0^1(\bar{\beta},\underline{\beta}))\right.$$

$$\left.+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\left(1+\frac{\xi}{(1+\lambda_f)(1-2\xi)}\right)\Phi(e_0^1(\bar{\beta},\underline{\beta}))\right)x_0^1(\bar{\beta},\underline{\beta})$$

$$+v(1-v)\left(\frac{\bar{S}+\underline{S}}{2}-(1+\lambda)(\underline{\beta}-e_0^2(\bar{\beta},\underline{\beta})+\psi(e_0^2(\bar{\beta},\underline{\beta})))\right)(1-x_0^1(\bar{\beta},\underline{\beta}))$$

$$+v(1-v)\left(\frac{\bar{S}+\underline{S}}{2}-(1+\lambda)(\underline{\beta}-e_0^1(\underline{\beta},\bar{\beta})+\psi(e_0^1(\underline{\beta},\bar{\beta})))x_0^1(\underline{\beta},\bar{\beta})\right)$$

$$+v(1-v)\left(\frac{\bar{S}+\underline{S}}{2}-(1+\lambda)(\bar{\beta}-e_0^2(\underline{\beta},\bar{\beta})+\psi(e_0^2(\underline{\beta},\bar{\beta})\right.$$

$$\left.+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\left(1+\frac{\xi}{(1-2\xi)(1+\lambda_f)}\right)\Phi(e_0^2(\underline{\beta},\bar{\beta}))\right)(1-x_0^1(\underline{\beta},\bar{\beta}))$$

$$+(1-v)^2\left(\frac{\bar{S}+\underline{S}}{2}-(1+\lambda)(\bar{\beta}-e_0^1(\bar{\beta},\bar{\beta})+\psi(e_0^1(\bar{\beta},\bar{\beta}))\right.$$

$$\left.+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\left(1+\frac{\xi}{(1-2\xi)(1+\lambda_f)}\right)\Phi(e_0^1(\bar{\beta},\bar{\beta}))\right)x_0^1(\bar{\beta},\bar{\beta}))$$

$$+(1-v)^2\left(\frac{\bar{S}+\underline{S}}{2}-(1+\lambda)(\bar{\beta}-e_0^2(\bar{\beta},\bar{\beta})+\psi(e_0^2(\bar{\beta},\bar{\beta}))\right.$$

$$\left.+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\left(1+\frac{\xi}{(1-2\xi)(1+\lambda_f)}\right)\Phi(e_0^2(\bar{\beta},\bar{\beta}))\right)(1-x_0^1(\bar{\beta},\bar{\beta})).$$

If $\sigma=1$,

$$e_1^1(\underline{\beta},\underline{\beta})=e^*=e_1^2(\underline{\beta},\underline{\beta}),$$

$$x_1^1(\underline{\beta},\underline{\beta})=1,$$

$$e_1^1(\bar{\beta},\underline{\beta})=\hat{e} \quad \text{and} \quad e_1^2(\bar{\beta},\underline{\beta})=e^*,$$

$$x_1^1(\bar{\beta},\underline{\beta})=1 \Leftrightarrow \Delta S-(1+\lambda)\,\Delta\beta>(1+\lambda)$$

$$\times\left\{\left(\psi(\hat{e})-\hat{e}+\frac{\lambda}{1+\lambda}\,\frac{v}{1-v}\,\Phi(\hat{e})\right)-(\psi(e^*)-e^*)\right\},$$

$$e_1^1(\underline{\beta},\bar{\beta})=e^* \quad \text{and} \quad e_1^2(\underline{\beta},\bar{\beta})=\hat{\hat{e}} \quad \text{and} \quad x_1^1(\underline{\beta},\bar{\beta})=1$$

with

$$\hat{\hat{e}}\in\arg\min_e\left\{\psi(e)-e+\frac{\lambda}{1+\lambda}\,\frac{v}{1-v}\,\frac{\lambda_f}{1+\lambda_f}\,\Phi(e)\right\},$$

$$e_1^1(\bar{\beta},\bar{\beta})=\hat{e}, \qquad e_1^2(\bar{\beta},\bar{\beta})=\hat{\hat{e}},$$

$$x_1^1(\bar{\beta},\bar{\beta})=1 \Leftrightarrow \Delta S>(1+\lambda)\left\{\left(\psi(\hat{e})-\hat{e}+\frac{\lambda}{1+\lambda}\,\frac{v}{1-v}\,\Phi(\hat{e})\right)\right.$$

$$\left.-\left(\psi(\hat{\hat{e}})-\hat{\hat{e}}+\frac{\lambda}{1+\lambda}\,\frac{v}{1-v}\,\frac{\lambda_f}{1+\lambda_f}\,\Phi(\hat{\hat{e}})\right)\right\}>0.$$

If $\sigma=2$, the solution is symmetric.

If $\sigma=0$,

$$e_0^2(\underline{\beta},\underline{\beta})=e_0^1(\underline{\beta},\underline{\beta})=e^*,$$

$$x_0^1(\underline{\beta},\underline{\beta})\in[0,1],$$

$$e_0^1(\bar{\beta},\underline{\beta})=\check{e},$$

with

$$\check{e}\in\arg\min_e\left\{\psi(e)-e+\frac{\lambda}{1+\lambda}\,\frac{v}{1-v}\left(1+\frac{\xi}{(1+\lambda_f)(1-2\xi)}\right)\Phi(e)\right\},$$

$$e_0^2(\bar{\beta},\underline{\beta})=e^* \quad \text{and} \quad x_0^1(\bar{\beta},\underline{\beta})=0,$$

$$e_0^1(\underline{\beta},\bar{\beta})=e^*;\; e_0^2(\underline{\beta},\bar{\beta})=\check{e} \quad \text{and} \quad x_0^1(\underline{\beta},\bar{\beta})=1,$$

$$e_0^1(\bar{\beta},\bar{\beta})=\check{e}=e_0^2(\bar{\beta},\bar{\beta}), \quad \text{and}$$

$$x_0^1(\bar{\beta},\bar{\beta}) \in [0,1].$$

From $A_1 = A_2 = 0$

$$s_1 = s^* + \frac{(1-v)}{(1+\lambda_f)}\left(x_0^2(\bar{\beta},\bar{\beta})\Phi(\breve{e}) - x_1^2(\bar{\beta},\bar{\beta})\Phi(\hat{\hat{e}})\right)$$

with $\breve{e} < \hat{\hat{e}}$

$$s_2 = s^* + \frac{(1-v)}{(1+\lambda_f)}\left(x_0^1(\bar{\beta},\bar{\beta})\Phi(\breve{e}) - x_2^1(\bar{\beta},\bar{\beta})\Phi(\hat{\hat{e}})\right).$$

As $x_0^2(\bar{\beta},\bar{\beta}) + x_0^1(\bar{\beta},\bar{\beta}) = 1$ and $\breve{e} < \hat{\hat{e}}$, the constraints $s_1 \geqq s^*$ and $s_2 \geqq s^*$ cannot hold unless

$$\Delta S > (1+\lambda)\Bigg[\left\{\psi(\hat{e}) - \hat{e} + \frac{\lambda}{1+\lambda}\frac{v}{1-v}\Phi(\hat{e})\right\}$$

$$-\left\{\psi(\hat{\hat{e}}) - \hat{\hat{e}} + \frac{\lambda}{1+\lambda}\frac{v}{1-v}\frac{\lambda_f}{1+\lambda_f}\Phi(\hat{\hat{e}})\right\}\Bigg], \tag{A.2.1}$$

that we refer to as case 1.

If (A.2.1) does not hold we have necessarily $s_1 = s_2 = s^*$. Then we must solve

$$\operatorname{Max} \xi W_1 + \xi W_2 + (1-2\xi)W_0 \quad \text{subject to}$$

$$(1-v)x_0^2(\bar{\beta},\bar{\beta})\Phi(e_0^2(\bar{\beta},\bar{\beta})) + v x_0^2(\underline{\beta},\bar{\beta})\Phi(e_0^2(\underline{\beta},\bar{\beta}))$$

$$-(1-v)x_1^2(\bar{\beta},\bar{\beta})\Phi(e_1^2(\bar{\beta},\bar{\beta})) - v x_1^2(\underline{\beta},\bar{\beta})\Phi(e_1^2(\underline{\beta},\bar{\beta})) \leqq 0,$$

$$(1-v)x_0^1(\bar{\beta},\bar{\beta})\Phi(e_0^1(\bar{\beta},\bar{\beta})) + v x_0^1(\bar{\beta},\underline{\beta})\Phi(e_0^1(\bar{\beta},\underline{\beta}))$$

$$-(1-v)x_2^1(\bar{\beta},\bar{\beta})\Phi(e_2^1(\bar{\beta},\bar{\beta})) - v x_2^1(\bar{\beta},\underline{\beta})\Phi(e_2^1(\bar{\beta},\underline{\beta})) \leqq 0.$$

Let μ_2 and μ_1 the Lagrange multipliers of the constraints $A_1 \geqq 0$ and $A_2 \geqq 0$. Clearly for both efficiency reasons and to weaken the collusion constraint $x_1^2(\underline{\beta},\bar{\beta}) = 0 = x_2^1(\bar{\beta},\underline{\beta})$. As $\mu_1 \geqq 0$, $\mu_2 \geqq 0$, we still have $x_0^1(\bar{\beta},\underline{\beta}) = 0$ and $x_0^2(\underline{\beta},\bar{\beta}) = 0$.

The constraints are reduced to

$$x_0^2(\bar{\beta},\bar{\beta})\Phi(e_0^2(\bar{\beta},\bar{\beta})) - x_1^2(\bar{\beta},\bar{\beta})\Phi(e_1^2(\bar{\beta},\bar{\beta})) \leqq 0,$$

$$x_0^1(\bar{\beta},\bar{\beta})\Phi(e_0^1(\bar{\beta},\bar{\beta})) - x_2^1(\bar{\beta},\bar{\beta})\Phi(e_2^1(\bar{\beta},\bar{\beta})) \leqq 0.$$

As $\mu_1 \geqq 0$, $\mu_2 \geqq 0$,

$$e_0^2(\bar{\beta},\bar{\beta}) \leqq e_1^2(\bar{\beta},\bar{\beta}),$$

$$e_0^1(\bar{\beta},\bar{\beta}) \leqq e_2^1(\bar{\beta},\bar{\beta}),$$

both constraints and lemma 1 can be satisfied only when

$$\Delta S=(1+\lambda)\left[\left\{\psi(\hat{e})-\hat{e}+\frac{\lambda}{1+\lambda}\frac{v}{1-v}\Phi(\hat{e})\right\}\right.$$

$$-\left\{\psi(\hat{\hat{e}}(\Delta S))-\hat{\hat{e}}(\Delta S)\right.$$

$$\left.\left.+\left(\frac{\lambda}{1+\lambda}\frac{v}{1-v}-\frac{\mu(\Delta S)}{(1-2\xi)(1-v)(1+\lambda)}\right)\Phi(\hat{\hat{e}}(\Delta S))\right\}\right], \tag{A.2.2}$$

where $\hat{\hat{e}}(\Delta S)$ is defined by:

$$\psi'(\hat{\hat{e}}(\Delta S))=1-\left(\frac{\lambda}{1+\lambda}\frac{v}{1-v}-\frac{\mu(\Delta S)}{(1-2\xi)(1-v)(1+\lambda)}\right)\Phi'(\hat{\hat{e}}(\Delta S)), \tag{A.2.3}$$

and $\mu(\Delta S)$ is the (symmetric) multiplier of the constraints $\{\hat{\hat{e}}(\Delta S), \mu(\Delta S)\}$ denotes the solution of (A.2.2) and (A.2.3); $\hat{e} \geqq \hat{\hat{e}}(\Delta S) \leqq \hat{e}$.

Then

$$x_0^2(\bar{\beta},\bar{\beta})=x_0^1(\bar{\beta},\bar{\beta})=1/2.$$

Let $\breve{e}(\Delta S)$ be the solution of

$$\psi'(\breve{e}(\Delta S))=1-\left(\frac{\lambda}{1+\lambda}\frac{v}{1-v}+\frac{\mu(\Delta S)}{(1-2\xi)(1-v)(1+\lambda)}\right)\Phi'(\breve{e}(\Delta S)).$$

We still have $\breve{e}(\Delta S) \leqq \hat{\hat{e}}(\Delta S)$. Choosing

$$x_1^2(\bar{\beta},\bar{\beta})=x_2^1(\bar{\beta},\bar{\beta})=\frac{1}{2}\frac{\Phi(\breve{e}(\Delta S))}{\Phi(\hat{\hat{e}}(\Delta S))}<1/2,$$

the collusion constraints are satisfied.

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